

BIR ZARRACHALI SHREDINGER OPERATORI XOS QIYMATI UCHUN ASSIMPTOTIK FORMULALAR

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ANNOTATSIYA

Ushbu ishda bir o'lchamli panjarada tashqi maydon ta'siri bilan kontakt ta'sirlashuvchi bir kvant zarracha harakatini tavsiflovchi bir zarrachali gamiltonianga mos bir zarrachali diskret Shredinger operatori h_μ ning zarrachalar ta'sirlashuv energiyasi $\mu > 0$ ga bog'liq chapda yoki o'ngda yagona xos qiymatga ega ekanligi isbotlangan.

Ushbu xos qiymat $z(\mu)$, $\mu > 1$ ning aniq ko'rinishi topilgan. O'zaro ta'sir parametri $\mu > 1$ ga monoton va uzluksiz bog'liqligi ko'rsatilib, moduli yetarlicha kichik $\mu > 1$ larda ushbu xos qiymatlar uchun yaqinlashuvchi yoyilmalar olingan.

***Kalit so'zlar:** panjara, operator, xos qiymat, spektr, muhim spektr, yoyilma.*

ASYMPTOTIC FORMULAS FOR EIGENVALUE OF ONE PARTICLE SCHRODINGER OPERATOR

ABSTRACT

In this study, one particle interaction energy of one particle discrete Schrödinger operator h_μ corresponding to a quantum particle gamiltonian describing the motion of a quantum particle in contact with an external field effect in a one-dimensional grid depends on $\mu > 0$ left or right proved to have a eigenvalue.

This eigenvalue $z(\mu)$, $\mu > 1$ a clear view of was found. The monotonic and continuous dependence of the interaction parameter on $\mu > 1$ is shown, and approximate distributions are obtained for these eigenvalues at sufficiently small modulus $\mu > 1$.

***Keywords:** lattince, operator, eigenvalue, spectrum, essential spectrum.*

KIRISH

Tashqi maydon ta'siridagi bir kvant zarracha harakatini tavsiflovchi bir zarrachali gamiltonianlar hamda qisqa masofada ta'sirlashuvchi ikkita bir xil va har xil zarrachalar sistemasi gamiltonianlariga mos ikki zarrachali diskret Shredinger

operatorlarining spektral xossalari Faria, Corolli, S.Albeverio, S.Laqayevlarning ishlarda o'rganilgan.

Ikki zarrachali diskret Shredinger operatori $h_\mu(0)$ bir zarrachali gamiltoniani h_μ ga unitar ekvivalent va shuning uchun bir zarrachali Shredinger operatorlarining spektral xossalari o'rganish panjaradagi ko'p zarrachali Shredinger operatorlar spektral nazariyasida muhim o'rin tutadi.

MUHOKAMA VA NATIJALAR

Ushbu [S.Lakaev, I.Bozorov] ishda uch o'lchamli panjarada potentsiilli maydonda tashqi maydon ta'sirida kontakt va bir qadamda ta'sirlashuvga mos bir zarrachali gamiltonian qaralgan. Unga mos diskret Shredinger operatori xos qiymatlar sonining zarrachaning ta'sir energiyalari $\mu > 0$ va $\lambda > 0$ larga bog'liqligi to'la o'rganilgan.

B.Saymon va M.Klausning ishida uzluksiz Shredinger operatori xos qiymatlari uchun kichik $\lambda > 0$ larda $d = 1$ da yaqinlashuvchi qator, $d = 2$ da esa kichik $\lambda > 0$ larda asimptotik yoyilma olingan.

2. Bir zarrachali Shredinger operatorining spektri

$\mathbb{T} = (-\pi, \pi]$ bir o'lchamli tor. $L_2(\mathbb{T})$ kvadrati bilan integrallanuvchi funksiyalar Gilbert fazosi. $L^t_2(\mathbb{T}) \subset L_2(\mathbb{T})$ toq funksiyalar qism fazosi bo'lsin.

$L^t_2(\mathbb{T})$ da aniqlangan quyidagi operatorni qaraymiz:

$$h_\mu = h_0 + \mu v,$$

Bu yerda h_0 operator $\varepsilon(t)$ funksiyaga ko'paytirish operatori, ya'ni

$$(h_0 f)(t) = \varepsilon(t) f(t);$$
$$\varepsilon(t) = 1 - \cos 2t;$$

ta'sir operatori (qo'zg'atish operatori) v operator esa integral operator bo'lib, $L^t_2(\mathbb{T})$ da quyidagicha aniqlangan:

$$(vf)(t) = \frac{1}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q \cdot f(q) dq;$$

$\mu > 0$ — zarrachaning tashqi muhit bilan ta'sir energiyasi.

Lemma 1. Yuqorida aniqlangan h_μ operator $L^t_2(\mathbb{T})$ da o'z-o'ziga qo'shma, chegaralangan operator bo'ladi.

Isbot. Hilbert fazosida o'z-o'ziga qo'shma operator ta'rifiga ko'ra $A: H \rightarrow H$ operator $\forall f, g \in H$ uchun $(Af, g) = (f, Ag)$ shartni qanoatlantirishi kerak. (\cdot, \cdot) — skalyar ko'paytma.

O`z-o`ziga qo`shma operator xossasiga ko`ra h_0 va v operatorlarning har birining o`z-o`ziga qo`shma operator ekanligini ko`rsatish yetarli.

$$\begin{aligned} (h_0 f, g) &= \int_{\mathbb{T}} (1 - \cos 2q) f(q) \overline{g(q)} dq = \int_{\mathbb{T}} f(q) (1 - \cos 2q) \overline{g(q)} dq = \\ &= \int_{\mathbb{T}} f(q) \overline{(1 - \cos 2q) g(q)} dq = (f, h_0 g); \end{aligned}$$

$$\begin{aligned} (v f, g) &= \int_{\mathbb{T}} \left(\frac{1}{2\pi} \sin 2q \int_{\mathbb{T}} \sin 2t f(t) dt \right) \cdot \overline{g(q)} dq = \\ &= \int_{\mathbb{T}} f(t) \left(\frac{1}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q \cdot g(q) dq \right) dt = (f, v g). \end{aligned}$$

Chegaralanganligi. Operator chegaralanganlik ta`rifidan A operator uchun $\exists M > 0$ soni mavjud bo`lib $\forall f \in D(A) \subset H$ element uchun quyidagi

$$\|(A f)\| \leq M \|f\|;$$

tengsizlik o`rinli bo`lsa, A operator chegaralangan deyiladi.

$$\begin{aligned} \|(h_{\mu} f)(t)\|^2 &= \int_{\mathbb{T}} \left| (1 - \cos 2t) f(t) + \frac{\mu}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 dt \leq \\ &\leq 2 \int_{\mathbb{T}} |(1 - \cos 2t) f(t)|^2 dt + 2 \int_{\mathbb{T}} \left| \frac{\mu}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 dt \\ &\leq 8 \int_{\mathbb{T}} |f(t)|^2 dt + 2 \int_{\mathbb{T}} \left| \frac{\mu}{2\pi} \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 dt = \\ &= 8 \int_{\mathbb{T}} |f(t)|^2 dt + \frac{\mu^2}{\pi} \left| \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2; \end{aligned}$$

Koshi – Bunyakovskiy tengsizligiga ko`ra:

$$\begin{aligned} \left| \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 &\leq \int_{\mathbb{T}} \sin^2 2q dq \cdot \int_{\mathbb{T}} |f(q)|^2 dq \\ &= \frac{1}{2} \int_{\mathbb{T}} (1 - \cos 2q) dq \cdot \int_{\mathbb{T}} |f(q)|^2 dq = \pi \|f\|^2; \end{aligned}$$

Demak, $\|(h_{\mu} f)(t)\| \leq \sqrt{8 + \pi} \|f\|$. Lemma isbotlandi. ▲

v – chekli oʻlchamli (bir oʻlchamli) operator boʻlganidan uning kompakt ekanligiga kelamiz.

h_μ operator oʻz-oʻziga qoʻshma boʻlganligi uchun uning qoldiq spektri boʻsh toʻplam boʻlib, spektri haqiqiy sonlar toʻplamining qismidan iborat, yaʼni

$$\sigma(h_\mu) \subset \mathbb{R}.$$

Quyidagi belgilashlarni kiritamiz:

$$\varepsilon_{\min} = \min_{q \in \mathbb{T}} \varepsilon(q) = 0; \quad \varepsilon_{\max} = \max_{q \in \mathbb{T}} \varepsilon(q) = 2;$$

Muhim spektr turgʻunligi haqidagi Veyl teoremasiga koʻra. h_μ operatorning muhim spektri $\sigma_{\text{ess}}(h_\mu)$ berilgan $\mu \geq 0$ parametrlardan bogʻliq emas va h_0 operatorning spektri bilan ustma-ust tushadi. Shunday qilib

$$\sigma_{\text{ess}}(h_\mu) = \sigma_{\text{ess}}(h_0) = \sigma(h_0) = [\varepsilon_{\min}, \varepsilon_{\max}] = [0, 2]$$

tenglik oʻrinli.

3. Asosiy natijalar bayoni.

Teorema 1; $\mu > 1$ boʻlsin h_μ muhim spektrdan oʻngda yagona xos qiymatga ega va u quyidagi koʻrinishga ega

$$z(\mu) = 2 + \frac{(\mu - 1)^2}{2\mu};$$

Bu xos qiymatga mos xos funksiyani koʻrinishi.

$$f(q) = -\frac{2\mu^2 C \sin 2q}{\mu^2 + 2\mu \cos 2q + 1}$$

dan iborat.

Natija. Quyidagi asimptotik formulalar oʻrinli

$$z(\mu) = 2 + \frac{1}{6}(\mu - 1)^2 + o(\mu - 1)^2$$

$$z(\mu) = \frac{\mu}{2} + o\left(\frac{1}{\mu}\right).$$

4. $\Delta_\mu(z)$ funksiyani xossalari.

C orqali kompleks sonlar tekisligini belgilaymiz, har bir $\mu > 0$ va $z \in C \setminus [0, 2]$ da aniqlangan

$$\Delta_\mu(z) = 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq; \quad (1)$$

funksiyani aniqlaymiz.

Bitiruv malakaviy ishini natijasini isbotlashda muhim o'rin tutadigan quyidagi lemmalarni isbotlaymiz.

Lemma 2. $z \in C \setminus [0,2]$ soni h_μ operatorning xos qiymati bo'lishi uchun

$$\Delta_\mu(z) = 0.$$

tenglikning bajarilishi zarur va yetarli.

Isbot; Zarurligi. $z \in C \setminus [0,2]$ soni h_μ operatorning xos qiymati bo'lsin. U holda

$$(\varepsilon(q) - z)f(q) = -\frac{\mu}{2\pi} \sin 2q \int_{\mathbb{T}} \sin 2t f(t) dt \quad (2)$$

tenglama nolmas $0 \neq f(q) \in L^t_2(\mathbb{T})$ yechimga ega.

Agar

$$\int_{\mathbb{T}} \sin 2t f(t) dt = C \quad (3)$$

deb belgilash kiritsak. ($C \neq 0$)

$$(\varepsilon(q) - z)f(q) = -\frac{\mu}{2\pi} \sin 2q C;$$

tenglikka kelamiz. Bundan $f(q)$ ni topamiz

$$f(q) = -\frac{\mu C}{2\pi} \frac{\sin 2q}{(\varepsilon(q) - z)}; \quad (4)$$

Ushbu (4) tenglik bilan aniqlangan $f(q)$ funksiyani (3) belgilashga keltirib qo'yamiz va quyidagi

$$C = -\frac{\mu C}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{(\varepsilon(q) - z)} dq$$

tenglikka ega bo'lamiz, ya'ni:

$$C + \frac{\mu C}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{(\varepsilon(q) - z)} dq = 0.$$

(1) belgilashga ko'ra

$$C \Delta_\mu(z) = 0.$$

$C \neq 0$ ekanligidan

$$\Delta_\mu(z) = 0$$

tenglikka kelamiz.

Yetarliligi. $\Delta_\mu(z) = 0$ bo'lsin. U holda $z \in C \setminus [0,2]$ soni h_μ operatorning xos qiymati bo'lishini ko'rsatamiz.

$$f(q) = -\frac{\mu C}{2\pi} \frac{\sin 2q}{(\varepsilon(q) - z)} = -\frac{\mu C}{2\pi} \frac{\sin 2q}{(1 - \cos 2q - z)};$$

funksiyani qurib olamiz. Dastlab $f(q) \in L_2^t(\mathbb{T})$ bo`lishini ko`rsatamiz.

$$\int_{\mathbb{T}} \left| -\frac{\mu C}{2\pi} \frac{\sin 2q}{(1 - \cos 2q - z)} \right|^2 dq = \left(\frac{\mu C}{2\pi}\right)^2 \int_{\mathbb{T}} \left(\frac{\sin 2q}{(1 - \cos 2q - z)}\right)^2 dq;$$

$z \in \mathbb{C} \setminus [0, 2]$ ekanligidan $1 - \cos 2q - z \neq 0$ va $\frac{\sin 2q}{(1 - \cos 2q - z)}$ — uzluksiz

funksiya. Bundan uning chegaralangan ekanligiga kelamiz. Demak, $f(q) \in L_2(\mathbb{T})$.

$\sin 2q$ — toq funksiya, $(1 - \cos 2q - z)$ — juft funksiya. Bundan $f(q)$ — toq funksiya. Demak, $f(q) \in L_2^t(\mathbb{T})$.

Endi bu funksiya z soni uchun (1) tenglamaning yechimi bo`lishini ko`rsatamiz.

$$\begin{aligned} &(\varepsilon(q) - z) \left(-\frac{\mu C}{2\pi} \frac{\sin 2q}{(1 - \cos 2q - z)}\right) \\ &= -\frac{\mu}{2\pi} \sin 2q \int_{\mathbb{T}} \sin 2t \left(-\frac{\mu C}{2\pi} \frac{\sin 2t}{(1 - \cos 2t - z)}\right) dt. \end{aligned}$$

Ushbu tenglikdan

$$\sin 2q + \frac{\mu}{2\pi} \sin 2q \int_{\mathbb{T}} \frac{\sin^2 2t}{(1 - \cos 2t - z)} dt = 0$$

ga kelamiz, ya`ni:

$$\sin 2q \left(1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2t}{(1 - \cos 2t - z)} dt\right) = 0;$$

(1) belgilashga ko`ra

$$\begin{aligned} \Delta_\mu(z) \sin 2q &= 0; \\ \Delta_\mu(z) &= 0. \end{aligned}$$

bundan (2) tenglikning o`rinli ekanligiga kelamiz.

Lemma isbot bo`ldi.

Lemma 2.3. $\Delta_\mu(z)$ funksiya $z \in (2; \infty)$ da monoton o`svuchi bo`lib

$$\begin{aligned} \lim_{z \rightarrow \infty} \Delta_\mu(z) &= 1; \\ \lim_{z \rightarrow 2} \Delta_\mu(z) &= 1 - \mu. \end{aligned}$$

Isbot; (1) formula bilan aniqlangan $\Delta_\mu(z)$ funksiyadan z bo`yicha hosila olamiz.

$$\frac{\partial \Delta_{\mu}(z)}{\partial z} = \mu \int_{\mathbb{T}} \frac{\sin^2 2q}{(\varepsilon(q) - z)^2} dq > 0.$$

Demak, $\Delta_{\mu}(z)$ funksiya monoton o'suvchi.

$$\frac{\sin^2 2q}{\varepsilon(q) - z}$$

funksiya $z > 2$ da uzluksiz va

$$\lim_{z \rightarrow \infty} \frac{\sin^2 2q}{\varepsilon(q) - z} = 0;$$

ekanligidan, integral belgisi ostida limitga o'tish haqidagi teorema ko'ra:

$$\lim_{z \rightarrow \infty} \Delta_{\mu}(z) = 1;$$

ga kelamiz.

$\sin^2 x + \cos^2 x = 1$ ayniyatga ko'ra:

$$\begin{aligned} \lim_{z \rightarrow 2} \Delta_{\mu}(z) &= \lim_{z \rightarrow 2} \left(1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq \right) = 1 + \frac{\mu}{2\pi} \lim_{z \rightarrow 2} \int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq = \\ &= 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \lim_{z \rightarrow 2} \frac{\sin^2 2q}{\varepsilon(q) - z} dq = 1 - \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{1 + \cos 2q} dq = \\ &= 1 - \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{1 - \cos^2 2q}{1 + \cos 2q} dq = 1 - \frac{\mu}{2\pi} \int_{\mathbb{T}} (1 - \cos 2q) dq = 1 - \mu. \end{aligned}$$

Lemma isbotlandi. ▲

Yuqoridagi lemmalardan kelib chiqadiki. $0 < \mu < 1$ bo'lsa

$$\Delta_{\mu}(z) = 0$$

tenglama yechimga ega emas. $\mu > 1$ da faqat bitta yechimga ega bo'ladi.

$$\mu = 1$$

bo'lsin.

$$\Delta_{\mu}(z) = 0;$$

tenglama $z = 2$ yechimga ega. $z = 2 \in \sigma_{ess}(h_{\mu})$. Bu son h_{μ} operatorning xos qiymat bo'lishini tekshiramiz.

$$(h_{\mu}f)(t) = 2f(t);$$

tenglamani qaraymiz. Bu tenglamadan

$$(1 + \cos 2t)f(t) = \frac{1}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q f(q) dq;$$

ga kelamiz.

$$\int_{\mathbb{T}} \sin 2q f(q) dq = C;$$

belgilash olamiz ($C \neq 0$). Bundan

$$f(t) = \frac{C}{2\pi} \cdot \frac{\sin 2t}{1 + \cos 2t};$$

$f(t) \neq 0$, endi $f(t) \in L_2^t(\mathbb{T})$ ni tekshiramiz.

$$\begin{aligned} \frac{4\pi^2}{C} \int_{\mathbb{T}} f^2(t) dt &= \int_{\mathbb{T}} \frac{\sin^2 2t}{(1 + \cos 2t)^2} dt = \int_{\mathbb{T}} \frac{1 - \cos 2t}{1 + \cos 2t} dt \\ &= \int_{-\pi}^0 \frac{1 - \cos 2t}{1 + \cos 2t} dt + \int_0^{\pi} \frac{1 - \cos 2t}{1 + \cos 2t} dt = 2 \int_0^{\pi} \frac{1 - \cos 2t}{1 + \cos 2t} dt \geq \end{aligned}$$

(bu xosmas integral bo`lib $t = \frac{\pi}{2}$ da maxsuslikka ega.)

$$\geq 2 \int_{\frac{\pi}{2}-\delta}^{\frac{\pi}{2}+\delta} \frac{1 - \cos 2t}{1 + \cos 2t} dt \geq C \int_{\frac{\pi}{2}-\delta}^{\frac{\pi}{2}+\delta} \frac{dt}{\left(t - \frac{\pi}{2}\right)^2} = C \int_{-\delta}^{\delta} \frac{dy}{y^2} = 2C_1 \int_0^{\delta} \frac{dy}{y^2}.$$

Ushbu munosabatning o`ng qismida turgan integral uzoqlashuvchi. Demak $f(t) \notin L_2^t(\mathbb{T})$. $z = 2$ xos qiymat emas.

Asosiy natijalar isboti

2.1 Teoremaning isboti.

Dastlab,

$$\int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq$$

integralni hisoblaymiz .

$$\int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq = \int_{\mathbb{T}} \frac{\sin^2 2q}{1 - \cos 2q - z} dq = \int_{\mathbb{T}} \frac{\sin^2 2q}{A - \cos 2q} dq;$$

bu yerda

$$A = 1 - z < -1;$$

Trigonometriyaning asosiy ayniyatidan foydalanib integralni ikki qismga ajratamiz.

$$\int_{\mathbb{T}} \frac{\sin^2 2q}{A - \cos 2q} dq = \int_{\mathbb{T}} \frac{1 - \cos^2 2q}{A - \cos 2q} dq = \int_{\mathbb{T}} \frac{dq}{A - \cos 2q} - \int_{\mathbb{T}} \frac{\cos^2 2q}{A - \cos 2q} dq$$

$$= I_1 + I_2; \quad (5)$$

Birinchi I_1 integralni hisoblaymiz:

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - \cos 2q};$$

Juft funksiyadan simmetrik oraliq bo'yicha olingan integral xossasidan foydalanib quyidagi tenglikka kelamiz:

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - \cos 2q} = \int_{-\pi}^0 \frac{dq}{A - \cos 2q} + \int_0^{\pi} \frac{dq}{A - \cos 2q} = 2 \int_0^{\pi} \frac{dq}{A - \cos 2q}.$$

Quyidagicha almashtirish olamiz

$$e^{2iq} = \xi;$$

bundan quyidagilar hosil bo'ladi:

$$\cos 2q = \frac{1}{2} \left(\xi + \frac{1}{\xi} \right),$$

$$d\xi = 2ie^{2iq} dq;$$

$$dq = \frac{d\xi}{2i\xi};$$

Topilganlarni integralga qo'yamiz.

$$I_1 = 2 \int_0^{\pi} \frac{dq}{A - \cos 2q} = 2 \int_{|\xi|=1} \frac{d\xi}{2i\xi \left(A - \frac{1}{2} \left(\xi + \frac{1}{\xi} \right) \right)} = \frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{2A\xi - \xi^2 - 1}$$

$$= -\frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{\xi^2 - 2A\xi + 1} = -\frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{(\xi - \xi_1)(\xi - \xi_2)};$$

bu yerda

$$\xi_1 = A - \sqrt{A^2 - 1};$$

$$\xi_2 = A + \sqrt{A^2 - 1};$$

Ko'rish mumkinki

$$|\xi_1| > 1, |\xi_2| < 1.$$

Koshining integral formulasi yordamida integralni hisoblaymiz.

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - \cos 2q} = -\frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{(\xi - \xi_1)(\xi - \xi_2)} = -\frac{2}{i} \cdot 2\pi i \cdot \frac{1}{\xi_2 - \xi_1}$$

$$= -\frac{2\pi}{\sqrt{A^2 - 1}};$$

Demak,

$$I_1 = -\frac{2\pi}{\sqrt{A^2 - 1}}. \quad (6)$$

Endi I_2 integralni hisoblaymiz.

$$I_2 = -\int_{\mathbb{T}} \frac{\cos^2 2q}{A - \cos 2q} dq; \quad (7)$$

Integral ostidagi kasr ifoda suratiga A^2 ni qo`shib ayiramiz va integralni quyidagi ikkita qismga ajratamiz.

$$I_2 = -\int_{\mathbb{T}} \frac{\cos^2 2q}{A - \cos 2q} dq = \int_{\mathbb{T}} \frac{A^2 - \cos^2 2q - A^2}{A - \cos 2q} dq$$

$$= \int_{\mathbb{T}} (A + \cos 2q) dq - A^2 \int_{\mathbb{T}} \frac{dq}{A - \cos 2q} = 2\pi A - A^2 I_1;$$

Demak,

$$I_2 = -\int_{\mathbb{T}} \frac{\cos^2 2q}{A - \cos 2q} dq = 2\pi A + \frac{2\pi A^2}{\sqrt{A^2 - 1}};$$

Lemma 2.4. ga ko`ra $z \in \mathbb{C} \setminus [0,2]$ soni h_μ operatorning xos qiymati bo`lishi uchun $\Delta_\mu(z) = 0$ tenglamani qanoatlantirishi zarur va yetarli. Faraz qilamiz $z \in \mathbb{C} \setminus [0,2]$ xos qiymat bo`lsin. U holda

$$\Delta_\mu(z) = 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq = 0;$$

ga ega bo`lamiz. (5), (6) va (7) tengliklarga ko`ra.

$$1 + \frac{\mu}{2\pi} \left(-\frac{2\pi}{\sqrt{A^2 - 1}} + 2\pi A + \frac{2\pi A^2}{\sqrt{A^2 - 1}} \right) = 0;$$

Ushbu tenglik

$$1 + \mu \left(A + \frac{A^2 - 1}{\sqrt{A^2 - 1}} \right) = 0$$

ga ekvivalent. Bundan

$$1 + \mu(A + \sqrt{A^2 - 1}) = 0;$$

$$\mu\sqrt{A^2 - 1} = -1 - \mu A;$$

$$(A < -1, \mu > 1 \text{ bo`lganidan } -1 - \mu A > 0)$$

Tenglamani ikkala tomonini kvadratga ko'taramiz.

$$\mu^2 A^2 - \mu^2 = 1 + 2\mu A + \mu^2 A^2.$$

O'xshash hadlarni qisqartirib soddalashtiramiz, natijada quyidagi A noma'lumga nisbatan chiziqli tenglamani hosil qilamiz:

$$1 + 2\mu A + \mu^2 = 0;$$

$A = 1 - z$ ekanligidan, quyidagi tenglikka kelamiz:

$$2\mu z = 1 + 2\mu + \mu^2;$$

$$z = 2 + \frac{1 - 2\mu + \mu^2}{2\mu};$$

yoki

$$z = 2 + \frac{(1 - \mu)^2}{2\mu}. \quad (8)$$

Bu xos qiymatga mos xos funksiyani aniqlaymiz. (3)ga ko'ra

$$f(q) = \frac{\mu C \sin 2t}{(1 - \cos 2q - z)}$$

Demak, (8) ni hisobga olsak bu xos qiymatga mos xos funksiya:

$$f(q) = -\frac{2\mu^2 C \sin 2q}{\mu^2 + 2\mu \cos 2q + 1}.$$

Ko'rinishda bo'lib bunda $C = \text{const} \neq 0$.

Teorema isbotlandi. ▲

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