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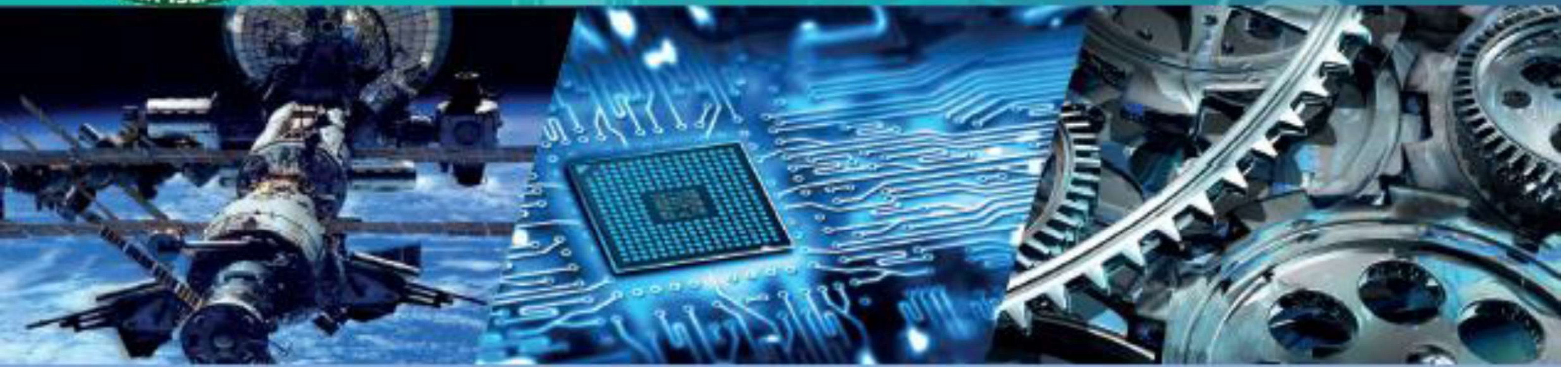

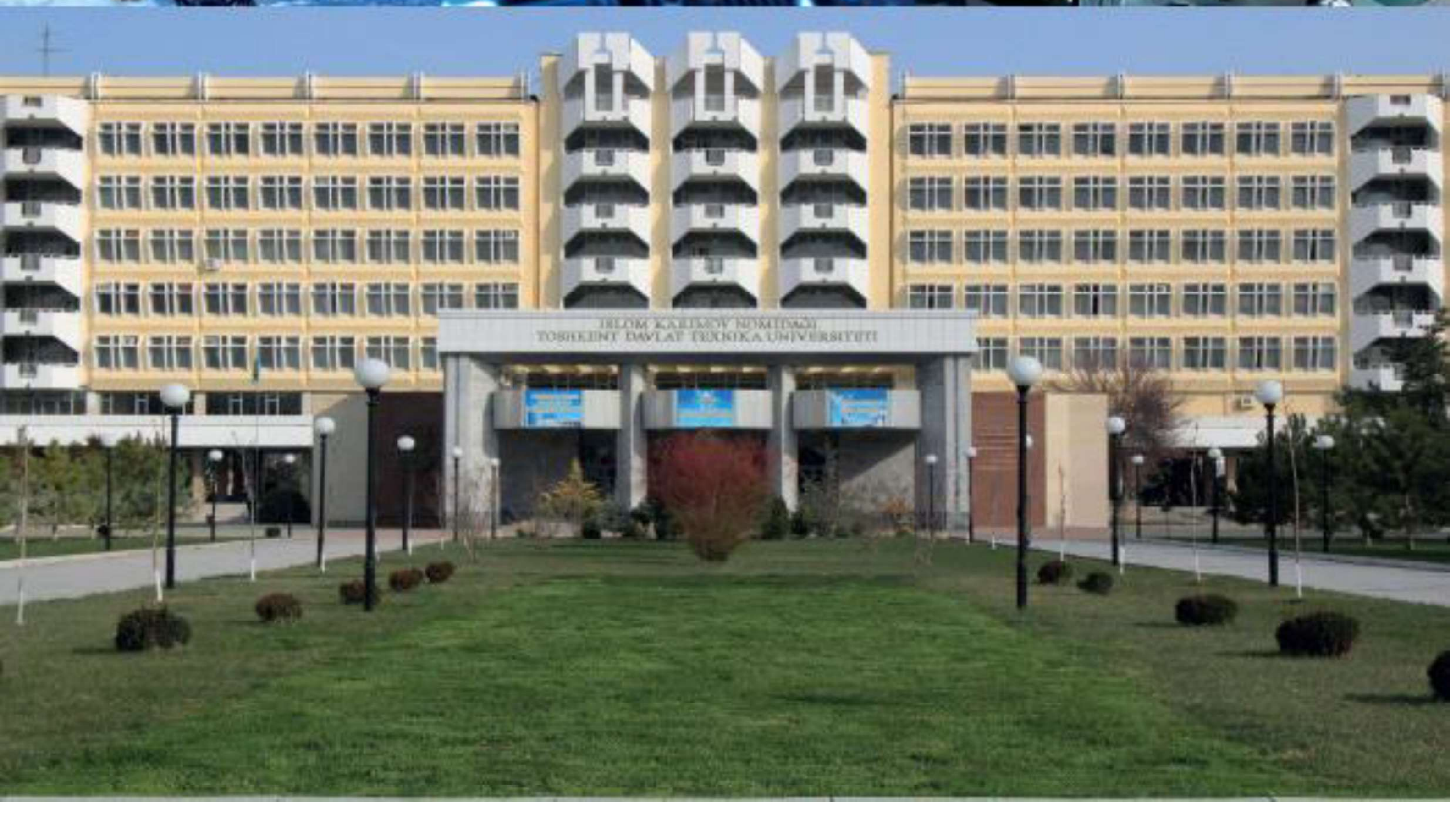

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A STABLE ITERATIVE ALGORITHM FOR ESTIMATING THE ELEMENTS OF THE MATRIX GAIN OF A KALMAN FILTER

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Abstract: *A stable iterative algorithm for estimating elements of the matrix gain of the Kalman filter has been developed. The traditional Kalman filter equations are given. Algorithms for autonomous calculation of the stationary Kalman filter gain are presented, which are performed under conditions relating to the system parameters. A non-linear iterative equation is solved for the gain of the Kalman filter. Modeling results are given, these Kalman filtering expressions for a linear discrete system and the actual filtering process is the current process for predicting and correcting recursive and iterative nature.*

Keywords: *dynamic object control systems, iterative algorithm, adaptive filtering, Kalman filter, estimation, covariance matrix, Kalman filter gain, modeling.*

INTRODUCTION. In most control processes or multi-step decision-making procedures in technical and technological systems, there are inherent uncertainties. The quality criterion is taken as the expectation of the cost function: it must be minimized using a sequence of controls. The procedure for choosing these controls, which is a multi-step decision-making process, is the problem of stochastic control [1-6].

The main reason for the divergence in the Kalman filter is that the filter gain tends to zero very quickly. Therefore, the estimate ceases to be dependent on the sequence of observations and the growing error of observations does not affect it. This difficulty can be overcome by one of the simple modifications of the filtering algorithm. Divergence may arise due to too little weight of new information or, conversely, due to too much weight of past observations. Therefore, divergence is a contradictory phenomenon. Thus, for example, when the input noise is small compared to the observations, the Kalman filtering error variance, and hence the gain, tends to decrease rapidly as time increases. In the case when the message model contains no control noise at all, the gain asymptotically tends to zero.

MATERIAL AND METHODS. This feature, which is not generally unexpected since each sample (on average) contains much more unwanted observation noise than control noise information, tends to "cut off" the filter from the observation sequence and thus cause instability. How strongly the divergence manifests itself and in what cases it will arise at all, all this depends on the length of the observation interval and on the accuracy of modeling the real process [1-6].

Thus, the noted circumstances indicate the need to create stable algorithms for estimating the state of dynamic systems under parametric a priori uncertainty and to synthesize computational schemes for their practical implementation [7-9]. Consider a linear dynamic system described by the equations:

$$\begin{aligned} x_{i+1} &= Ax_i + w_i, \\ z_i &= Hx_i + v_i, \quad i \geq 0, \end{aligned} \tag{1}$$

where x_i is the n-dimensional state vector at time i, z_i is the m-dimensional measurement vector at time i, A is the nxn system transition matrix, H is the mxn output matrix, w_i is the plant noise at time i, and v_i is the measurement noise at time i. $\{w_i\}$ and $\{v_i\}$ are Gaussian zero-mean white random processes with covariance matrices Q and R, respectively.

If we talk about the Kalman filter, the discrete time Kalman filter [8,10,11] is the most well-known algorithm that solves the filtering problem. In fact, Kalman filter faces simultaneously two problems as follows: estimation (the aim is to recover at time *i* information about the state vector at time *i* using measurements up till time *i*) and prediction (the aim is to obtain at time *i* information about the state vector at time *i* +1 using measurements up till time *i*; it is clear that prediction is related to the forecasting side of information processing).

To estimate the state vector x_i of the dynamic system (1), traditional Kalman filter equations of the form are usually used:

$$K_i = P_{i/i-1} H^T (H P_{i/i-1} H^T + R)^{-1}, \tag{2}$$

$$P_{i/i} = (I - K_i H) P_{i/i-1}, \tag{3}$$

$$P_{i+1/i} = Q + A P_{i/i} A^T, \tag{4}$$

where, for $i \geq 0$, with initial condition $P_{0/-1} = P_0$ for the time instant, where there are no measurements given, The Kalman filter gain K_i is a matrix of dimension *nxm*, *R* and P_0 are positive definite matrices. We present algorithms for the steady state Kalman filter gain autonomous computation. These algorithms hold under conditions concerning the system parameters. We define the matrix:

$$G_i = K_i H, \tag{5}$$

where G_i is a nonsymmetric matrix of dimension *nxn*. Information for us, it is also clear that there exists a steady state value:

$$\widetilde{G} = \widetilde{K} H. \tag{6}$$

Also, we define the matrix:

$$S = H^T R^{-1} H. \tag{7}$$

Note that *S* is an *nxn* symmetric positive semidefinite matrix and *S* is a positive definite if *rank(H)=n*; this means that *S* is a nonsingular matrix in the case, *rank(H)=n* with *m>n* [12]. Now, the Kalman direct steady state filter gain factors are calculated. We present algorithms for the direct computation of the steady state Kalman filter $\widetilde{K}$. The proposed algorithms compute directly the steady state Kalman filter gain, that is, without using $\widetilde{G} = \widetilde{K} H$. All these algorithms hold under the assumption that *n=m*. Note that, since $rank(H) = n$, *H* and *S* are nonsingular matrices. On a basis (5) and (7), we are able to derive the following nonlinear iterative equation with respect to the Kalman filter gain K_k:

$$K_{k+1} = G_{k+1} H^{-1} = (QA^{-T} H^T R^{-1} + AK_k)[HQA^{-T} H^T R^{-1} + RH^{-T} A^{-T} H^T R^{-1} + HAK_k]^{-1}, \tag{8}$$
$$G_{i+1} = ((QA^{-T} S) + (A) G_i)[(S^{-1} A^{-T} S + QA^{-T} S) + (A) G_i]^{-1}.$$

Then, the nonsingularity of *S* and (7) allow us to write the equality in (8) as:

$$K_{k+1} = (QA^{-T}H^TR^{-1} + AK_k)[HQA^{-T}H^TR^{-1} + HS^{-1}A^{-T}H^TR^{-1} + HAK_k]^{-1} = \\ = (C + DK_k)[L + BK_k]^{-1}, \tag{9}$$

Where

$$L = HQA^{-T}H^TR^{-1} + HS^{-1}A^{-T}H^TR^{-1} = H(Q + S^{-1})A^{-T}H^TR^{-1},$$
$$B = HA,$$
$$C = QA^{-T}H^TR^{-1}, \tag{10}$$

RESULTS AND DISCUSSION. Thus, it is known [10] that the prediction error covariance tends to the steady state prediction error covariance and that the convergence is independent of the initial uncertainty, that is, independent of the value of the initial condition P_0. Thus, we are able to assume zero initial condition $P_0=0$ and so we are to use the initial condition $K_0=0$. It is clear that K_k tends to a steady state value $\widetilde{K}$ satisfying: $\widetilde{K} = (C + D\widetilde{K})[L + B\widetilde{K}]^{-1}$.

Based on expressions (9), we are able to derive the following nonlinear iterative equation with respect to the Kalman filter gain K_k: $K_{k+1} = (C + DK_k)[L + BK_k]^{-1} = c + l[K_k^{-1} + b]^{-1}d$, where *L, B, C, D* are given by (10) and
$l = D - CL^{-1}B,\ b = L^{-1}B,\ c = CL^{-1},\ d = L^{-1},\ K_0 = P_0H^T[HP_0H^T + R]^{-1}$. It is known [7-10] that the prediction error covariance tends to the steady state prediction error covariance and that the convergence is independent of the initial uncertainty, that is, independent of the value of the initial condition P_0.

Thus, we are able to assume zero initial condition $P_0=0$. In this case, in order to avoid K_0^{-1}, we are to use the initial condition $K_1 = c$. It is clear that K_k tends to a steady state value $\widetilde{K}$ satisfying: $\widetilde{K} = c + a[\widetilde{K}^{-1} + b]^{-1}d$. It follows from this that, we can consider problems of doubling the iterative algorithm. In (9), setting: $K_k = Y_kX_k^{-1}$,:
$Y_{k+1}X_{k+1}^{-1} = (C + DY_kX_k^{-1})[L + BY_kX_k^{-1}]^{-1} = (CX_k + DY_k)[LX_k + BY_k]^{-1}$, $\begin{bmatrix} X_{k+1} \\ Y_{k+1} \end{bmatrix} = \Phi \begin{bmatrix} X_k \\ Y_k \end{bmatrix}$, where $F = \begin{bmatrix} L & B \\ C & D \end{bmatrix} = \begin{bmatrix} d^{-1} & d^{-1}b \\ cd^{-1} & cd^{-1}b + l \end{bmatrix}$ is a matrix of dimension $2n \times 2n$ and *L, B, C, D* as in (10).

Thus, working as in the doubling iterative algorithm and using zero initial condition $P_0 = 0$, so $K_0 = Y_0X_0^{-1} = 0$; we are able to derive the following nonlinear iterative equations:

$$l_{k+1} = l_k(I - c_k[I + b_kc_k]^{-1}b_k)l_k,$$
$$b_{k+1} = b_k + d_k[I + b_kc_k]^{-1}b_kl_k,$$
$$c_{k+1} = c_k + l_kc_k[I + b_kc_k]^{-1}d_k,$$
$$d_{k+1} = d_k[I + b_kc_k]^{-1}d_k,$$
$$l_1 = l,\ b_1 = b,\ c_1 = c,\ d_1 = d.$$

It is clear that $c_k = Y_{2^k} X_{2^k}^{-1} = K_{2^k}$ tends to a steady state value $\widetilde{K}$. Using on this iterative algorithm, we will create an algebraic algorithm for $\widetilde{K}$ computation. Working as in the algebraic algorithm and using the parameters *L, B, C, D* by (10), we derive

$F = \begin{bmatrix} L & B \\ C & D \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda^{-1} \end{bmatrix} \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}^{-1}$, which is a matrix of dimension $2n \times 2n$. Then, the steady state Kalman filter is $\widetilde{K} = W_{21} W_{11}^{-1}$.

This algorithm for the computation of the steady state Kalman filter gain $\widetilde{K}$. It is clear that the direct computation of the Kalman filter gain is feasible only if the following restriction holds: *n=m*. The advantage of the presented algorithms is the autonomous computation of the steady state Kalman filter gain. Especially, the steady state Kalman filter gain is important, when we want to compute the parameters of the steady state Kalman filter:

$$x_{k+1/k+1} = (I - \widetilde{K}H) A x_{k/k} + \widetilde{K} z_{k+1} = (I - \widetilde{G}) A x_{k/k} + \widetilde{K} z_{k+1}. \tag{11}$$

It is obvious from (11) that the parameters of the steady state Kalman filter are related to the steady state Kalman filter gain. In particular, the steady state prediction error covariance can be computed via the steady state gain and is given by

$$\widetilde{P}_p = [I - \widetilde{K}H]^{-1} \widetilde{K} R H [H^T H]^{-1} \ . \tag{12}$$

Indeed, from (2), arises $\widetilde{K} = \widetilde{P}_p H^T [H \widetilde{P}_p H^T + R]^{-1}$, which leads to

$$\widetilde{K}(H\widetilde{P}_p H^T + R) = \widetilde{P}_p H^T \Rightarrow \widetilde{K} H \widetilde{P}_p H^T + \widetilde{K} R = \widetilde{P}_p H^T$$
$$\Rightarrow \widetilde{P}_p H^T - \widetilde{K} H \widetilde{P}_p H^T = \widetilde{K} R \Rightarrow (I - \widetilde{K}H) \widetilde{P}_p H^T = \widetilde{K} R$$
$$\Rightarrow \widetilde{P}_p H^T = [I - \widetilde{K}H]^{-1} \widetilde{K} R \Rightarrow \widetilde{P}_p H^T H = [I - \widetilde{K}H]^{-1} \widetilde{K} R H.$$

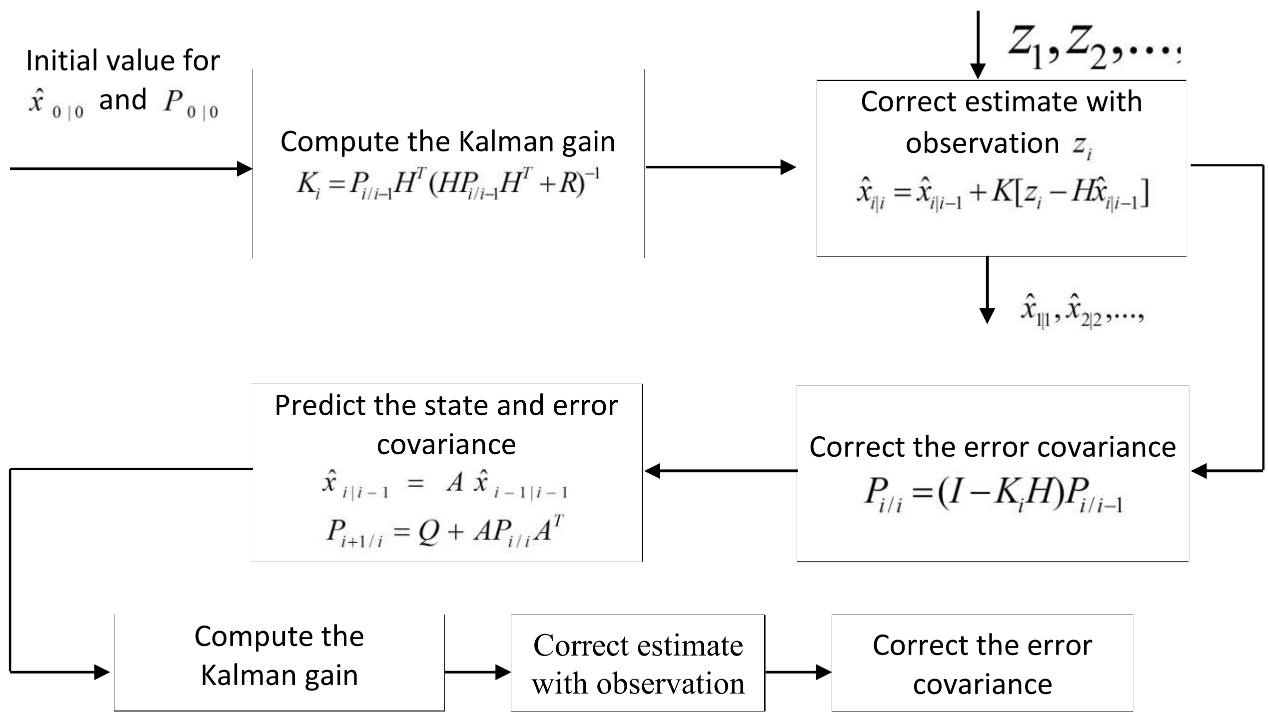

Fig 1. Computing circuit based on the Kalman filter.

Based on the above, we can say, the Kalman filter gain arises in Kalman filter equations in linear estimation and is associated with linear systems. The gain is a matrix through which the estimation and the prediction of the state as well as the corresponding estimation and prediction error covariance matrices are computed. For time invariant and asymptotically stable systems, there exist steady state values of the estimation and prediction error covariance matrices. There exists also a steady state value of the Kalman filter gain.

Iterative algorithms as well as an algebraic algorithm for the steady state Kalman filter computation were presented. These algorithms hold under conditions concerning the system parameters. The advantage of these algorithms is the autonomous computation of the steady state Kalman filter gain.

CONCLUSION. Thus, if we talk about modeling, these Kalman filtering expressions for a linear discrete system and the actual filtering process is the current process for predicting and correcting recursive and iterative nature [13-15]. This does not require storing large amounts of data. When new data is received, the new filtering value can be calculated at any time. The figure shows the operating mode of the calculator based on the Kalman filter. This stable iterative algorithm for estimating elements of the matrix gain of the Kalman filter allows stabilizing the synthesis of the dynamic object control system

References:

1. A.A. Krasovsky. Handbook on the theory of automatic control. *"Science",* Moscow, **1987**. 712.
2. V.N. Afanasyev. Dynamic control systems with incomplete information: Algorithmic construction. *"Publishing House",* **2007**.
3. S.V. Pervachev., A.I. Perov. Adaptive message filtering. *"Radio and Communications",* Moscow, **1991**. 160.
4. M.A. Ogarkov. Methods for statistical evaluation of random process parameters. *"Energo atom publishing",* Moscow, **1990**. 208.
5. H.Z. Igamberdiyev., A.N. Yusupbekov., O.O. Zaripov., J.U. Sevinov. Algorithms of adaptive identification of uncertain operated objects in dynamical models. *"Procedia Computer Science",* **2017**. 854.
6. J.U. Sevinov., O.O. Zaripov., S.O. Zaripova. The algorithm of adaptive estimation in the synthesis of the dynamic objects control systems. *"International journal of advanced science and technology",* **2020**. 1096.
7. N. Assimakis., M. Adam. "Kalman filter Riccati equation for the prediction, estimation and smoothing error covariance matrices. *"ISRN Computational Mathematics",* **2013**. 7.
8. N. Assimakis., M. Adam. Global systems for mobile position tracking using Kalman and Lainiotis filters. *"The Scientific World Journal",* **2014**. 8.
9. N. Assimakis., M. Adam. Iterative and algebraic algorithms for the computation of the steady state kalman filter gain. *"Hindawi publishing corporation, applied mathematics",* **2014**. 10.
10. B.D.O. Anderson., J.B. Moore. *"Optimal filtering, dover publications"*, USA, **2005**.
11. M.S. Grewal., A.P. Andrews. *"Kalman filtering: theory and practice using MATLAB",* USA, **2008**.
12. R.A. Horn., C.R. Johnson. Matrix analysis, *"Cambridge university press",* Cambridge, UK, **2005**.
13. F.P. Vasiliev. Optimization methods. *"Publishing House: Factorial Press",* **2002**. 824.
14. A.B. Bakushinsky., M.Y. Kokurin. *"Iterative methods of solving irregular equations",* Moscow, **2006**. 214.
15. A.Y. Morozov., D.L. Reviznikov. *"Methods of computer simulation of dynamic systems with interval parameters",* Moscow, **2019**.

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